# On the Existence of Leaky Waves Due to a Line Source Above a Grounded Dielectric Slab*

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*Summary*—**The existence of a particular type of leaky wave is verified experimentally. The leaky wave considered is that due to an electric line source above a dielectric slab. Since such a leaky wave cannot exist by itself, it must be detected in the presence of the remainder of the continuous spectrum and often in the presence of a surface wave (or waves). This was done by probing the fields and observing an interference pattern between the leaky wave and the existing surface wave, as predicted by the theory. These results emphasize a need to take leaky waves into account in the design of surface wave components and antennas.**

## INTRODUCTION

A LEAKY wave is broadly characterized as a wave which has a continuous leakage of energy away from the surface with which its propagation is associated. The most familiar case in which this arises is that of a long slot perturbing a normally closed waveguide.

A more careful enumeration of the characteristics of the leaky waves puts them in sharp contrast to surface waves, with which there has been considerable confusion in the past. A leaky wave is not a proper modal solution, whereas the surface wave is. The surface wave propagates unattenuated parallel to the guiding surface and has pure attenuation transverse to the surface. The leaky wave, in contrast, has complex propagation constants in both longitudinal and transverse directions, attenuating in the forward phase direction along the surface and growing transverse to the surface. The surface wave is a slow wave, whereas the leaky wave has a longitudinal phase velocity greater than that of light. Finally, the surface wave cannot radiate, whereas the leaky wave is not bound to the surface and may therefore contribute to the radiation field of the structure.

Although, in general, leaky waves exist on structures that are quite different from those supporting surface waves, there are some structures that support both types of waves. The existence of leaky waves on a surface interface structure has been postulated by Zucker.[1] It was subsequently found that, in the absence of sources at finite distances, leaky waves would exist in a physical sense only if the guiding structure were modulated periodically.[2-4]

In the presence of a source, it has been found that leaky waves also exist on unmodulated structures. Marcuvitz[5] has predicted the existence of nonmodal waves due to a source exciting a uniform guiding surface, which have characteristics identical in form to the waves referred to above. Marcuvitz[5] points out that such waves are, in reality, part of the continuous spatial spectrum of the source and are distinct from the discrete (modal) spectrum of surface waves. Furthermore, these waves are exponentially attenuated along any radius from the source within their domain of existence and therefore comprise a part of the near field, so to speak, of the source.

Interest in such waves should not be limited to the theoretical since the design of excitation structures for surface-wave transmission lines, antennas and cavities should take these waves into account if there are obstructions to or changes in the surface-wave structure anywhere but at great distances from the source (or launcher).

A particular case of leaky waves due to sources has been worked out in detail by Barone.[6] Barone has found the resonances, both proper and improper, for the case of an electric line source over a dielectric slab and has made numerical calculations for several typical cases of parameters.

The present paper takes the odd solutions of Barone (the "short circuit bisection"), which apply to the case of a grounded dielectric slab, and reports experimental verification of his results through use of the physically realizable dielectric loaded trough waveguide. The dielectric loaded trough waveguide, with spacing be-

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[1] F. J. Zucker, "The guiding and radiation of surface waves," *Proc. Symp. on Modern Advances in Microwave Techniques*, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., November 8–10, 1954, vol. 4, pp. 403–435; November, 1954.

[2] A. S. Thomas and F. J. Zucker, "Radiation from modulated surface wave structures—I," 1957 IRE NATIONAL CONVENTION RECORD, pt. 1, pp. 153–160.

[3] R. L. Pease, "Radiation from modulated surface wave structures," 1957 IRE NATIONAL CONVENTION RECORD, pt. 1, pp. 161–165.

[4] A. A. Oliner and A. Hessel, "Guided waves on sinusoidally-modulated reactance surfaces," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-7, pp. S201–S208; December, 1959.

[5] N. Marcuvitz, "On field representations in terms of leaky modes or eigen modes," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-4, pp. 192–194; July, 1956.

[6] S. Barone, "Leaky Wave Contributions to the Field of a Line Source Above a Dielectric Slab," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., Rept. R-532-56, PIB-462; November, 1956.

tween side walls small compared to the wavelength, supports TE waves which are the sole wave type excited in the infinite configuration. The spacing between side walls may be made small enough so that a single conductor, fed by a coaxial line through one wall, will approximate the uniform line source to any degree of accuracy desired.

The results of the present work show the correspondence of experimental data with calculations for one of the cases of Barone. The most striking aspect of the results is an interference pattern, primarily between surface wave and leaky wave, existing out along the line several wavelengths from the source.

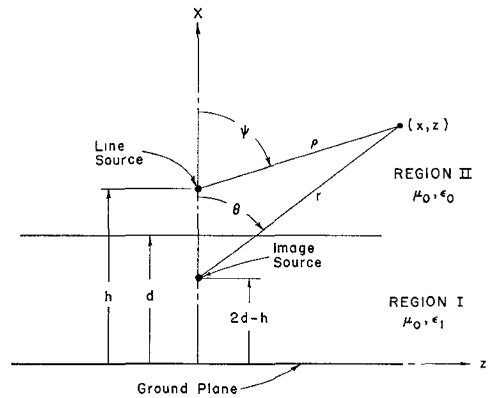


Fig. 1—End view of the line source over a grounded dielectric slab.

## Theoretical Background

The structure being investigated and the coordinate system used are shown in Fig. 1. The expression for the total electric field in Region II ($E_{y2}$) due to an electric line source over a grounded dielectric slab has been found previously[6,7] and may be written as follows:

$$E_{y2}(x, z) = \frac{i\omega\mu_0}{2\pi}\int_{-\infty}^{\infty}\frac{1}{-2i\kappa} \cdot \left[e^{i\kappa|x-h|} + \Gamma e^{i\kappa(x+h-2d)}\right]e^{i\zeta z}d\zeta, \quad (1)$$

where

$$\Gamma = \frac{\kappa - i\kappa_\epsilon \cot \kappa_\epsilon d}{\kappa + i\kappa_\epsilon \cot \kappa_\epsilon d}, \quad (2)$$

$$\kappa = \sqrt{k^2 - \zeta^2}, \quad (3)$$

$$\kappa_\epsilon = \sqrt{k^2 K_1 - \zeta^2}, \quad (4)$$

$$k^2 = \omega^2\mu_0\epsilon_0 = (2\pi/\lambda_0)^2, \qquad K_1 = \epsilon_1/\epsilon_0.$$

In (1), the integral has been written so that the first term in the bracket may be seen to be the field due to the line source alone. We shall now show, following Barone, that the total field can be expressed as the sum of three types of terms: leaky-wave terms, surface-wave terms, and radiative terms. The radiative terms will be found to represent the far fields of the source and its image in the dielectric. Eq. (1) may be rewritten in the following form:[6]

$$E_{y2}(r, \theta) = \frac{-\omega\mu_0}{4}H_0^{(1)}(k\rho) - \frac{\omega\mu_0}{4\pi}\int_{-\infty}^{\infty}\Gamma e^{i\kappa(x+h-2d)+i\zeta z}d\zeta, \quad (5)$$

where

$x-h=\rho\cos\psi$ and $z=\rho\sin\psi$,

$H_0^{(1)}$ is the Hankel function of zero order and first kind.

The second part of this expression evidently is due to an image source of strength $\Gamma$ at $x=2d-h$, $z=0$. The two parts of this expression may also be called the "incident wave" and the "reflected wave," respectively, having reference to the transverse transmission line analysis of Barone.[6]

[7] C. T. Tai, "The effect of a grounded slab on the radiation from a line source," *J. Appl. Phys.*, vol. 22, pp. 405–414; April, 1951.

The integrand of the second part of (5) has a branch point at $\zeta=k$ and an infinite number of poles. The poles, some pure real and others complex, have been found graphically for parameters of the geometry (Fig. 1) and the dielectric constant by Barone.[6] In the original contour of integration of Tai,[7] it would be found that there are residues due to the real poles and a contribution due to integration around the branch cut, but the complex poles are not included within the contour. The real-pole residues are surface-wave contributions and the branch-cut contribution may be called the "entire space-wave" contribution. The latter is discussed more fully below.

The branch-cut contribution is integrated by using steepest-descent methods, which yields the familiar radiation terms. Prior to performing the steepest-descent integration, the following change of variables is made:

$$\zeta = k\sin\phi, \qquad \phi = \xi + i\eta. \quad (6)$$

When the path of integration is deformed to the path of steepest descent, it is found that some of the complex poles are now included within the contour. Residues due to the complex poles have been shown by Barone[6] to be of the leaky-wave form,

$$E_y^L(r, \theta) = Ae^{\alpha k_\theta' r}e^{i\beta k_r' r}, \quad (7)$$

where

$A$ = a constant specified by $h$, $d$, and $K_1$,
$\alpha = \sinh\eta'$,
$\beta = \cosh\eta'$,
$k_r' = k\cos(\theta-\xi')$,
$k_\theta' = -k\sin(\theta-\xi')$,
$x+h-2d = r\cos\theta$,
$z = r\sin\theta$,
$\phi' = \xi' + i\eta'$ = location of the complex pole in the $\phi$ plane.

It should be emphasized that these leaky-wave contributions are merely a part of the particular field representation of the entire space wave chosen by using the steepest-descent method of integration and are to be included with the radiation and higher-order terms obtained from the saddle point or steepest-descent integration.

The surface-wave contributions (*i.e.*, residues of real poles) have been indicated by Barone.[6] Complete data on the lower-order surface waves of the trough line has been given by Cohn.[8,9] The excitation of the lowest order of these modes by a current element in the physical situation, depicted in Fig. 1, has been studied by Cohn, Cassedy and Kott.[10] The expression for this lowest-order mode (the $TE_{20}$ mode, in Cohn's designation) may be written as follows:[9,10]

$$E_y{}^s(x, z) = \omega\mu_0 \frac{(k_{10}d)\sin^2 k_{10}d}{(\zeta_p d)(\tan k_{10}d - k_{10}d)} e^{-k_{20}(h-2d+x)+j\zeta_p z}, \quad (8)$$

where

$$\zeta_p = \sqrt{\omega^2\mu_0\epsilon_1 - k_{10}{}^2} = \sqrt{\omega^2\mu_0\epsilon_0 + k_{20}{}^2}$$

= values of the propagation constant at the pole.

The values of $k_{10}d$ and $k_{20}d$ are given by Cohn[8,9] for various $K_1$.

The steepest-descent evaluation of the second part of (5) yields an asymptotic series in "$kr$," the leading term of which is:

$$f(r, \theta) = \frac{i}{4\pi}\sqrt{\frac{\pi}{kr}}\, G(\theta) e^{ikr} \quad (9)$$

where

$$G(\theta) = \frac{\cos\theta - i\sqrt{K_1 - \sin^2\theta}\cot\left[kd\sqrt{K_1 - \sin^2\theta}\right]}{\cos\theta + i\sqrt{K_1 - \sin^2\theta}\cot\left[kd\sqrt{K_1 - \sin^2\theta}\right]}.$$

Higher-order terms of the expansion are readily evaluated[11] and will be mentioned below.

The addition of (9) and the first term of the (Hankel type) asymptotic expansion of the line source yields

$$E_y{}^R(x, z) = -\frac{\omega\mu_0}{4}\sqrt{\frac{2}{\pi}}\frac{(i)^{-1/2}}{(k\rho)^{1/2}} e^{ik\rho} + \omega\mu_0 f(r, \theta), \quad (10)$$

where $r$, $\theta$ and $\rho$ were given previously as functions of $x$, $y$. This may be termed the radiative term of the entire space wave of the structure.

The entire space wave, which includes all saddle-point terms and leaky-wave terms given in this representation, has been shown to be orthogonal to each of the surface-wave terms,[12] but no orthogonality exists between individual terms of the space wave. The entire space wave, arising from the branch-cut contribution, is seen to be a continuous spectrum of spatial frequencies.[5,12] The surface-wave terms correspond to discrete spatial frequencies, obeying all orthogonality requirements of proper modes. The leaky waves, on the other hand, do not have these properties and are thus termed "improper modes."[5,12]

## Calculations

One of the complex poles found by Barone[13] corresponded to $2d/\lambda = 1.08$, $h/d = 1.10$ and $K_1 = 2.56$. The computation of the residue for that pole then completes the data necessary to specify the constant $A$ in (7). Barone[6] has shown that this is by far the strongest leaky wave present for these physical parameters, due to the greater rate of attenuation of all others.

It also may be seen that only one surface wave exists[6,8,9] for the above-specified parameters. The expression (8) for this mode is completely specified for the above parameters.

In order to specify the total electric field for the above parameters, we need to add the fields of the leaky waves, the surface waves, the line source [first term of (5)] and the complete asymptotic expansion for the saddle-point contribution of the image source [second part of (5)]. For the computation we start with the first terms only of these space-wave terms (10), yielding the following approximate expression for the total field:

$$E_{y2}(x, z) \simeq E_y{}^L(x, z) + E_y{}^S(x, z) + E_y{}^R(x, z). \quad (11)$$

It was found that a particularly striking manner to present the computed results, was to make plots of $E_y(x, z)$ vs $z$ for parameters of constant $x$. This method of presentation shows marked interference patterns in the range

$$2 < z/\lambda_0 < 6, \qquad 0.6 < x/\lambda_0 < 0.75.$$

This interference, it turns out, is primarily between surface wave and leaky wave, with $E_y{}^R(x, z)$ modifying the result only slightly. Furthermore, investigation of the second-order terms of the primary and image source space waves show them to be at least an order of magnitude smaller than the sum of the three terms of (11) everywhere in the above specified range. The second-order terms were therefore not included in the data presented below. The results are shown in Fig. 2.

Fig. 3 shows a plot of constant amplitude lines for each of the three components of (11) for the case $2d/\lambda_0 = 1.08$, $h/d = 1.10$ and $K_1 = 2.56$. The region in the neighborhood of the critical angle[14] is not accurate since there the saddle-point contribution is modified by the proximity of the complex pole to the path of steepest

[8] M. Cohn, "Propagation in a dielectric loaded parallel plane waveguide," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 202–208; April, 1959.

[9] M. Cohn, "TE modes of the dielectric loaded trough line," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 449–454; July, 1960.

[10] M. Cohn, E. S. Cassedy, and M. A. Kott, "TE mode excitation on dielectric loaded parallel plane and trough waveguides," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8; pp. 545–552; September, 1960.

[11] L. Felsen and N. Marcuvitz, "Modal Analysis and Synthesis of Electromagnetic Fields," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., Rept. R776-59; October, 1959.

[12] L. Felsen and N. Marcuvitz, "Modal Analysis and Synthesis of Electromagnetic Fields," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., Rept. R726-59; June, 1959.

[13] $N = 3$ resonance of Barone.[6]

[14] It is shown by Barone[6] that the leaky wave does not exist above a certain critical angle "$\theta_c$." Briefly, this is due to the path of steepest descent's being a function of the angle of observation ($\theta$). Only for angles of $\theta_c$ and greater is the leaky-wave pole captured within the closed path of integration and, therefore, add its residue to the solution.

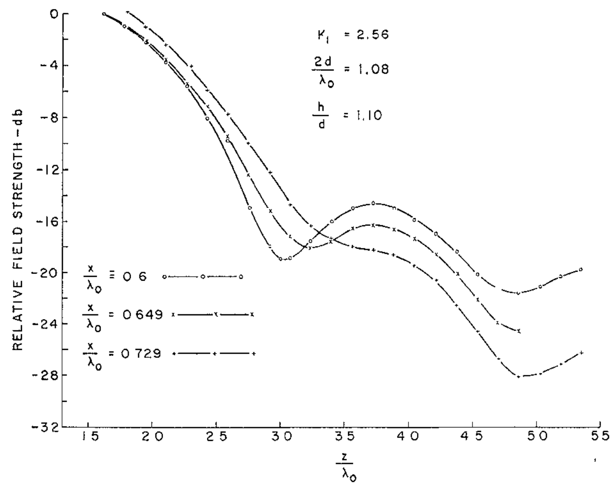


Fig. 2—Calculated curves of the total electric field vs normalized horizontal distance from the line source ($z/\lambda_0$) for various values of normalized distance above the ground plane ($x/\lambda_0$).

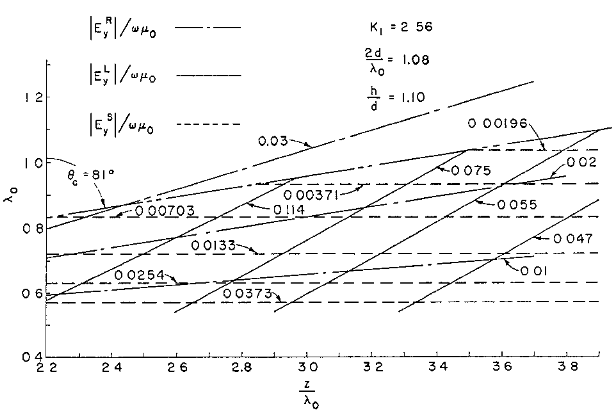


Fig. 3—Calculated constant amplitude lines of radiation field $|E_y^R|$, leaky-wave field $|E_y^L|$, and surface-wave field $|E_y^S|$.

descents.[11] The necessary correction to the saddle-point contribution has not been made here. This correction should make the field everywhere continuous in the region of the critical angle to satisfy the physical requirement of continuity.

## Measurements and Apparatus

As stated in the Introduction, the mathematical formulation of this problem assumes a physically unrealizable geometry and excitation condition. If two parallel conducting planes, spaced a distance "$b$" apart, are located on the above structure so as to be perpendicular to the current source, then the physically realizable dielectric loaded trough waveguide results. The fields which would exist on the unrealizable structure of Fig. 1 are undisturbed by the addition of the two conducting planes, but the total power emanating from the unit current source is now finite.

The trough waveguide, in which the fields were explored, and the method of exciting the current source have been previously reported.[9,10,15] The measurements reported in this paper were made at a frequency of 4.86 Gc. The trough line and current source are specified by the following parameters: $K_1 = 2.56$, $d = 3.33$ cm, $b = 0.79$ cm, and $h = 3.70$ cm. The geometric parameters were those existing on the trough line previously built to determine the properties of the $TE_{20}$ mode and to measure the efficiency with which the $TE_{20}$ mode could be launched from a current source.[9,10] The frequency was chosen to provide a value of $(2d/\lambda_0)$, which yielded a large amplitude leaky wave as determined from Barone's computations.[6]

Matched loads were placed along the top of the trough line and on one side of the current source in order to provide reflectionless terminations for both the radiating fields and any waves which have a component of propagation in the negative $z$ direction. These loads consisted of tapered sections of white pine wood, which were impregnated with a liquid containing suspended carbon.

The probe used to sample the fields in the interior of the trough guide is shown in Fig. 4. This probe is held

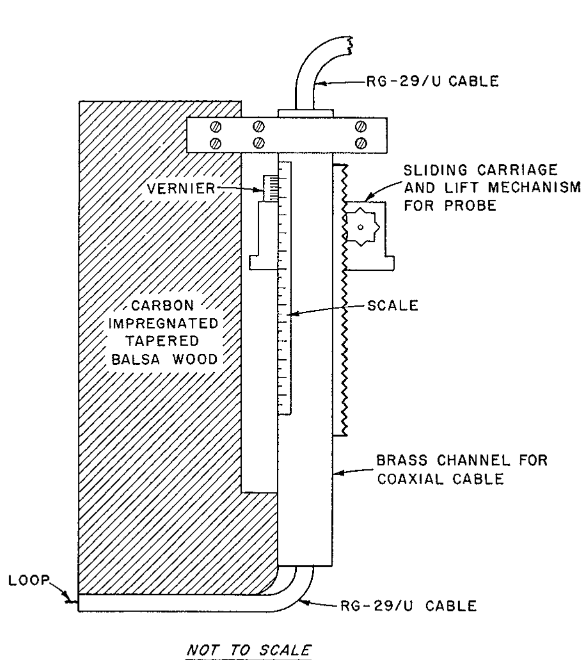


Fig. 4—Side view of the probe used to sample the fields in the trough line.

in place by a carriage assembly which in turn slides along the top edges of the side walls of the trough. The vertical position of the probe can be changed by the rack and pinion arrangement shown. In order to insure accurate motion of the probe, the top edges of the side walls were carefully machined to be parallel to the inside of the bottom of the trough. Scales with 1-mm graduations and verniers were mounted on the probe and trough line so that horizontal and vertical positions of the probe could be read to an accuracy of 0.1 mm.

The probe consists of a length of RG-29/U coaxial cable terminated by a small loop. This cable runs through a rectangular cross-section brass block and around the underside of a matched load, with only the loop extending ahead of the matched load. The entire

[15] M. Cohn, "Propagation in Partially Dielectric Loaded Parallel Plane and Trough Waveguides," Radiation Lab., The Johns Hopkins University, Baltimore, Md., Tech. Rept. No. AF-78; July, 1960.

probe assembly thus appears to be a reflectionless termination to the impinging wave. The loop was formed by bending the extended center conductor of the cable into a circle of approximately 1/8 inch diameter and connecting it to the cable's outer conductor. This loop lies in a horizontal plane ($x$ = constant) and couples to the $x$ component of the magnetic field.

The probe was set at a fixed height above the bottom of the trough guide ($x/\lambda_0 = 0.60$) and the relative field strength was measured as a function of distance along the guide from the current source ($z/\lambda_0$). This procedure was repeated for two other probe heights ($x/\lambda_0 = 0.649$ and $x/\lambda_0 = 0.729$) to determine if the theoretically predicted field distributions of Fig. 2 existed. The results of these measurements are shown in Fig. 5. The predicted interference between the leaky and surface waves, as manifested by the decrease in field strength near the predicted values of ($z/\lambda_0$) is clearly shown. Possible causes of the discrepancy in the depths of the measured nulls compared to the predicted nulls will be discussed below.

The probe was also used in conjunction with a phase-sensitive bridge to determine constant phase contours in the vicinity of the first amplitude null ($2.3 < z/\lambda_0 < 3.2$). The measured constant phase contours are shown in Fig. 6. It should be noted, from the calculated constant amplitude surfaces of Fig. 3, that the constant phase contours of Fig. 6 were measured in a region where the leaky wave is the principal contributor to the total field. The slope of these constant phase contours is in good agreement with the calculated slope of the leaky wave phase fronts. The center phase front, having the large wiggles, is the one which occurs at the null of the interference pattern. It is hence the least accurate, since the field amplitude is very low at that point. There is no discontinuity of the phase fronts at the critical angle ($\theta = \theta_c$), but rather there is a smooth blending of the phase fronts of the leaky wave with the radiation field.

Also measured but not shown in this paper were the constant amplitude contours in the region where the leaky wave is the principal wave. The slope of these measured constant amplitude contours was in good agreement with the calculated leaky-wave amplitude contours of Fig. 3. The change of amplitude between adjacent contours was also in good agreement with the predicted values.

There are a number of possible sources of error in the above measurements. One such source of error is due to reflected waves coming from the certainly imperfect matched loads at the top of the trough line and on the negative $z$ side of the current source. The error caused by these reflections would probably be greatest in the vicinity of the amplitude nulls. Another source of error is the low signal-to-noise ratio obtainable near the amplitude nulls. The ratio of source height to dielectric slab thickness ($h/d$) used for the measurements was 1.11 rather than 1.10 as used for the calculations. A further source of error is that the normalized spacing between the parallel walls of the trough guide ($b/\lambda_0 = 0.128$), though small, is not negligible.

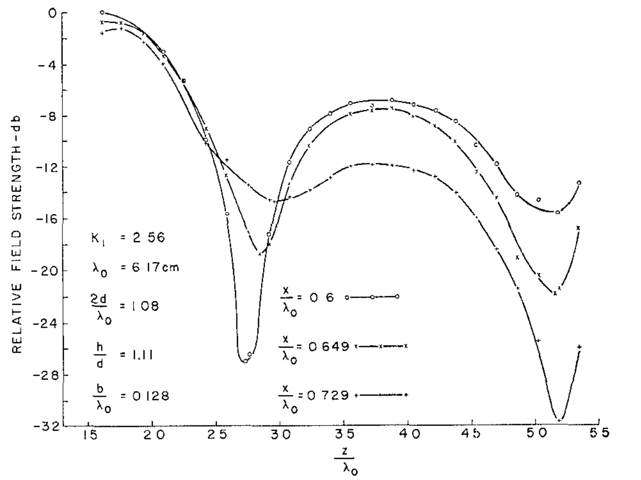


Fig. 5—Measured values of the total electric field in the trough line vs normalized horizontal distance from line source ($z/\lambda_0$) for various values of normalized distance above ground plane ($x/\lambda_0$).

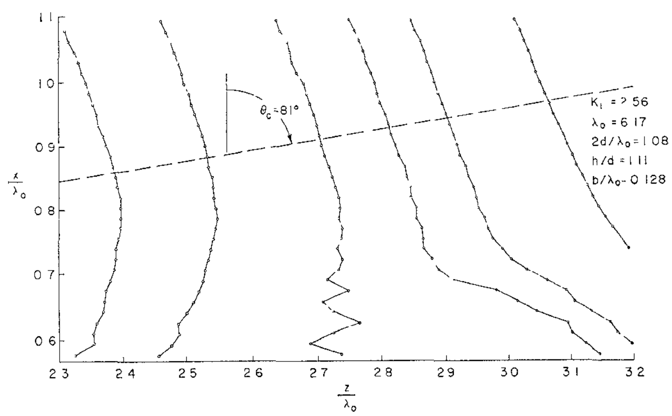


Fig. 6—Measured constant phase contours in the trough line.

## Conclusions

Since a leaky wave can never exist by itself, but must be detected in the presence of the remainder of the continuous spatial spectrum and often in the presence of surface waves as well, it is difficult to measure its characteristics. The measurements reported above, however, confirm the existence and predicted characteristics of leaky waves due to a line current source above a grounded dielectric slab.

The predicted and subsequently measured interference between the leaky wave and surface wave strongly emphasizes the need to take leaky waves into account in the design of surface waveguide components and antennas. The particular surface-wave structure and excitation configuration should be investigated to determine if leaky waves exist and what effect they may have on the field distribution.

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